Long-Term Obligations

Part 2

Your goals for this "long-term obligations" chapter are to learn about:

- Long-term notes and present value concepts.
- The nature of bonds and related terminology.
- Accounting for bonds payable, whether issued at par, a premium or discount.
- Effective-interest amortization methods.
- Special considerations for bonds issued between interest dates and for bond retirements.
- Analysis, commitments, alternative financing arrangements, leases, and fair value measurement.

6. Long-Term Notes

The previous chapter illustrations of notes were based on the assumption that the notes were of fairly short duration. Now, let's turn our attention to longer term notes. A borrower may desire a longer term for their loan. It would not be uncommon to find two, three, five-year, and even longer term notes. These notes may evidence a "term loan," where "interest only" is paid during the period of borrowing and the balance of the note is due at maturity. The entries are virtually the same as you saw in the previous chapter. As a refresher, assume that Wilson issued a five-year, 8% term note – with interest paid annually on September 30 of each year:

10-1-X3	Cash		10,000	
	Note Payable			10,000
	To record note payable at 8% per annum; maturity date on 9-30-X8			
12-31-XX	Interest Expense		200	
	Interest Payable			200
	To record accrued interest for 3 months ($10,000 X 8% X 3/12) at end of each year			
9-30-XX	Interest Expense		600	
	Interest Payable		200	
	Cash			800
	To record interest payment ($10,000 X 8% = $800, of which $200 was previously accrued at the prior year end) each September			
9-30-X8	Interest Expense		600	
	Interest Payable		200	
	Note Payable		10,000	
	Cash			10,800
	To record final interest payment and balance of note at maturity			

Other notes may require level payments over their terms, so that the interest and principal are fully paid by the end of their term. Such notes are very common. You may be familiar with this type of arrangement if you have financed a car or home. By the way, when you finance real estate, payment of the note is usually secured by the property being financed (if you don't pay, the lender can foreclose on the real estate and take it over). Notes thus secured are called "mortgage notes."

6.1 How do I Compute the Payment on a Note?

With the term note illustrated above, it was fairly easy to see that the interest amounted to $800 per year, and the full $10,000 balance was due at maturity. But, what if the goal is to come up with an equal annual payment that will pay all the interest and principal by the time the last payment is made? From my years of teaching, I know that students tend to perk up when this subject is covered. It seems to be a relevant question to many people, as this is the structure typically used for automobile and real estate ("mortgage") financing transactions. So, now you are about to learn how to calculate the correct amount of the payment on such a loan. The first step is to learn about future value and present value calculations.

6.2 Future Value

Let us begin by thinking about how invested money can grow with interest. What will be the future value of an investment? If you invest $1 for one year, at 10% interest per year, how much will you have at the end of the year? The answer, of course, is $1.10. This is calculated by multiplying the $1 by 10% ($1 X 10% = $0.10) and adding the $0.10 to the dollar you started with.
And, if the resulting $1.10 is invested for another year at 10%, how much will you have? The answer is $1.21. That is, $1.10 X 10% = $0.11, which is added to the $1.10 you started with. This process will continue, year after year. The annual interest each year is larger than the year before because of "compounding." Compounding simply means that your investment is growing with accumulated interest, and you are earning interest on previously accrued interest that becomes part of your total investment pool. In contrast to "compound interest" is "simple interest" that does not provide for compounding, such that $1 invested for two years at 10% would only grow to $1.20.
Not to belabor the mathematics of the above observation, but you should note the following formula:

$$(1+i)^n$$

Where "i" is the interest rate per period and "n" is the number of periods

The formula will reveal how much an investment of $1 will grow to after "n" periods. For example, $(1.10)^2 = 1.21$. Or, if $1 was invested for 5 years at 6%, then it would grow to about $1.34 ($(1.06)^5 = 1.33823$). Of course, if $1,000 was invested for 5 years at 6%, it would grow to $1,338.23; this is determined by multiplying the derived factor times the amount invested at the beginning of the 5-year period. Hopefully, you will see that it is not a great challenge to figure out how much an up-front lump sum investment can grow to become after a given number of periods at a stated interest rate. This calculation is aptly termed the "future value of a lump sum amount." Future Value Tables are available that include precalculated values (the tables are found in the Appendix to this book). See if you can find the 1.33823 factor in a future value table. Likewise, use the table to determine that $5,000, invested for 10 years, at 4%, will grow to $7,401.20 ($5,000 X 1.48024).

6.3 Present Value

Present value is the opposite of future value, as it reveals how much a dollar to be received in the future is worth today. The math is simply the reciprocal of future value calculations:

$$1/(1+i)^{n}$$

Where "i" is the interest rate per period and "n" is the number of periods

For example, $1,000 to be received in 5 years, when the interest rate is 7%, is presently worth $712.99 ($1,000 X (1/(1.07)^5). Stated differently, if $712.99 is invested today, it will grow to $1,000 in 5 years. Present Value Tables are available in the appendix. Use the table to find the present value of $50,000 to be received in 8 years at 8%; it is $27,013.50 ($50,000 X .54027).

6.4 Annuities

Streams of level (i.e., the same amount each period) payments occurring on regular intervals are termed "annuities." For example, if you were to invest $1 at the beginning of each year at 5% per annum, after 5 years you would have $5.80. This amount can be painstakingly calculated by summing the future value amount associated with each individual payment, as shown at below.

Year of Investment	Future Value Factor From Table	Payment	Value of Payment at End of 5th Year
1 (amount will be invested 5 years)	1.27628	$1	$1.27628
2 (amount will be invested 4 years)	1.21551	$1	$1.21551
3 (amount will be invested 3 years)	1.15763	$1	$1.15763
4 (amount will be invested 2 years)	1.10250	$1	$1.10250
5 (amount will be invested 1 year)	1.05000	$1	$1.05000
			$5.80192

But, it is much easier to use to an Annuity Future Value Table. The annuity table is simply the summation of individual factors. You will find the "5.80191 " factor in the 5% column, 5 year row. These calculations are useful in financial planning. For example, you may wish to have a target amount accumulated by a certain age, such as with a retirement contribution account. These tables will help you calculate the amount you need to set aside each period to reach your goal. See the book Appendix for this table.

Conversely, you may be interested in an Annuity Present Value Table. This table (which is simply the summation of amounts from the lump sum present value table - with occasional rounding) shows factors that can be used to calculate the present worth of a level stream of payments to be received at the end of each period. This table is found in the Appendix to the book. Can you use the table to find the present value of $1,000 to be received at the end of each year for 5 years, if the interest rate is 8% per year, is $3,992.71? Look at the 5 year row, 8% column and you will see the 3.99271 factor.

6.5 Returning to the Original Question

How do you compute the payment on a typical loan that involves even periodic payments, with the final payment extinguishing the remaining balance due? The answer to this question is found in the present value of annuity calculations. Remember that an annuity involves a stream of level payments, just like many loans. Now, think of the payments on a loan as a series of level payments that covers both the principal and interest. The present value of those payments is the amount you borrowed, in essence removing ("discounting") out the interest component. This may still be a bit abstract, and can be further clarified with some equations. You know the following to be true for an annuity:

Present Value of Annuity = Payments X Annuity Present Value Factor

A loan that is paid off with a series of equal payments is also an annuity, therefore:

Loan Amount = Payments X Annuity Present Value Factor

Thus, to determine the annual payment to satisfy a $100,000, 5-year loan at 6% per annum:

$100,000 = Payment X 4.21236 (from table)
Payment = $100,000/4.21236
Payment = $23,739.64

You can safely conclude that 5 payments of $23,739.64 will exactly pay off the $100,000 loan and all interest. Simply stated, the payments on a loan are just the loan amount divided by the appropriate present value factor. To fully and finally prove this point, let's look at a typical loan amortization table. This table will show how each payment goes to pay the accumulated interest for the period, and reduce the principal, such that the final payment will pay the remaining interest and principal. You should study this table carefully:

Yr	Beginning of Year Loan Balance	Interest on Beginning Balance	Amount of Payment	Principal Reduction (payment minus interest)	End of Year Loan Balance
1	$100,000.00	$6,000 ($100,000.00 X 6%)	$23,739.64	$17,739.64 ($23,739.64 - $6,000.00)	$82,260.36 ($100,000.00 - $17,739.64)
2	$82,260.36	$4,935.62 ($82,260.36 X 6%)	$23,739.64	$18,804.02 ($23,739.64 - $4,935.62)	$63,456.34 ($82,260.36 - $18,804.02)
3	$63,456.34	$3,807.38 ($63,456.34 X 6%)	$23,739.64	$19,932.26 ($23,739.64 - $3,807.38)	$43,524.08 ($63,456.34 - $19,932.26)
4	$43,524.08	$2,611.44 ($43,524.08 X 6%)	$23,739.64	$21,128.20 ($23,739.64 - $2,611.44)	$22,395.89 ($43,524.38 - $21,128.20)
5	$22,395.89	$1,343.75 ($22,395.89 X 6%)	$23,739.64	$22,395.89 ($23,739.64 - $1,343.75)	$0 ($22,395.89 - $22,395.89)

The journal entries associated with the above loan would flow as follows:

1-1-X1	Cash		Pennies 100,000	
	Note Payable			100,000.00
	To record note payable			
12-31-X1	Interest Expense		6,000.00	
	Note Payable		17,739.64	
	Cash			23,739.64
	To record interest payment			
12-31-X2	Interest Expense		4,935.62	
	Note Payable		18,804.02	
	Cash			23,739.64
	To record interest payment			
12-31-X3	Interest Expense		3,807.38	
	Note Payable		19,932.26	
	Cash			23,739.64
	To record interest payment			
12-31-X4	Interest Expense		2,611.44	
	Note Payable		21,128.20	
	Cash			23,739.64
	To record interest payment			
12-31-X5	Interest Expense		1,343.75	
	Note Payable		22,395.89	
	Cash			23,739.64
	To record interest payment			

6.6 A Few Final Comments on Future and Present Value

- Be very careful in performing annuity related calculations, as some scenarios may involve payments at the beginning of each period (as with the future value illustration above, and the accompanying future value tables), while other scenarios will entail end-of-period payments (as with the note illustration, and the accompanying present value table). In later chapters of this book, you will be exposed to additional future and present value tables and calculations for alternatively timed payment streams (e.g., present value of an annuity with payments at the beginning of each period).

- Payments may occur on other than an annual basis. For example, a $10,000, 8% per annum loan, may involve quarterly payments over two years. The quarterly payment would be $1,365.10 ($10,000/7.32548). The 7.32548 present value factor is reflective of 8 periods (four quarters per year for two years) and 2% interest per period (8% per annum divided by four quarters per year). This type of modification does not only pertain to annuities, but also to lump sums. For example, the present value of $1 invested for five years at 10% compounded semiannually can be determined by referring to the 5% column, ten-period row.

- Numerous calculators include future and present value functions. If you have such a machine, you should become familiar with the specifics of its operation. Likewise, spreadsheet software normally includes embedded functions to help with fundamental present value, future value, and payment calculations. Following is a screen shot of one such routine: